‘‘Modelling Optimal Strategies for Deteriorating Inventory Systems under Different Scenarios’’

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1. Abstract

Product deterioration, a genuine phenomenon, poses an obstacle to numerous inventory management systems. The shelf life of a product and its physical condition is expressed by its deterioration rate. An extremely fascinating aspect of managing inventory is mathematical modelling for products that are deteriorating. The pace of deterioration varies depending on the product's characteristics or service. The deterioration behaviour for an inventory model can belong to different kinds: the models with a fixed lifetime of the product, the models with an age-dependent deterioration rate, i.e., a probabilistic distributed life time, and the models with a time- or stock-based or constant deterioration rate. *Our research carried out three different scenarios for deteriorating inventory models: scenario 1: the inventory models of the "new and buyback used products" concept; scenario 2: the inventory models with carbon emissions and green investments; and scenario 3: the inventory models that incorporated freshness and greening efforts for perishable products.*

The purchasing behaviour of consumers has changed nowadays. Consumers not only prefer the newly launched product on the market, but they are also interested in purchasing used, refurnished, recycled, or repaired products with price discounts. They are also concerned about environmental issues and prefer to purchase goods from manufacturers or retailers with a green reputation. For this reason, a lot of businesses have started gathering used products that buyers throw away. With this in mind, we created deteriorating inventory models without and with shortages as retailer points for both newly released products and buyback used products and optimized the ordering quantity of new products, buyback quantity of used products, and replenishment cycle time such that the retailer's profit is maximized.

Controlling carbon emissions has been the primary objective for nations since the emission of carbon causes numerous problems in the global ecosystem. Ordering, production setup, purchasing, storage, impact on the environment, transport, and other inventory system activities all result in the emission of carbon. The management of deteriorating inventory with green technology investment has been one of the areas that contribute to mitigating carbon emissions. Our research work focused on designing sustainable inventory models to minimize carbon emissions using green investments and applying various carbon policies and trade credit payment systems to the demand of deteriorating products depending on

selling price, green investment cost, and their promotion, resulting in a total profit maximized and supply chain costs minimized.

Nowadays, consumers who are concerned about their health prefer and expect nutritional and fresh sustainable products. Product freshness is an essential component of its quality, and as a result, the choice of purchase for consumers depends on the freshness of the green products. Due to the effect of physical deterioration and quality degradation of the product; the product loses its originality continuously, so market demand decreases and hence retailer or producers offers the price discount or markdown strategy to stimulate the demand. Greening efforts are the action taken to minimize the impact trade has on the ecosystem and ensure sustainable products. Taking into account all of these factors, developed the inventory models with the demand is a function of the selling price, age of products (freshness), and greening efforts for deteriorating perishable products and optimize the retailer's or producer's profit is maximize.

The objective of the research work is to maximize the total profit or minimize the total cost of the retailer, producer, or manufacturer at the optimal value of decision variables. Models are validated through numerical examples, sensitivity analysis of parameters, and graphical demonstrations of objective functions, and managerial insights are derived from the analysis. Some concluding remarks, along with future scopes, are discussed in each chapter.

2. Brief description on the state of the art of the research topic

One of the most important aspects of supply chain management nowadays is inventory management, which has a significant impact on how well a business succeeds. Controlling the basic components necessary to produce products and finished products for sale is another aspect of effective inventory management. Inventory management can be applying through a mathematical modelling of inventory. An inventory model (EOQ/EPQ) is a mathematical tool that assists businesses to identify the optimum quantity of inventories that should be maintained in a production process, handling ordering frequency, deciding the number of products or raw materials to be preserved, and monitoring the rate of supply of products and supplies in order to ensure continuous service to consumers with no shipment failures. The development of economic ordering quantity model or economic production quantity model where need to considering different costs related to inventory system.

In the inventory management market demand factor play the major role. Demand distributions as deterministic or stochastic [1]. As per the literature deterministic demand can be classified as constant/uniform, price-dependent, time-varying, stock-dependent, combination of time, price, stock level, green investment dependent, credit period dependent, promotional level, etc. and stochastic demand can be categorised with known and arbitrary demand distribution. It is necessary to consider the characteristics of products and external factors in inventory management. As an illustration, managing perishable inventory and non-perishable inventory requires different management abilities because they have different characteristics.

Product deterioration is one of the characteristics that point need must attention for inventory management. The effect of deterioration and their mathematical modelling of inventory system first derived Ghare and Schrader (1963) [2]. Consumption of products that were deteriorating was strongly related to the exponential forms of time [2]. An up-to-date review on inventory systems for deteriorating items is presented by Shah and Shah (2000) [3], Goyal and Giri (2001) [4], Bakker et al. (2012) [5], and Janssen et al. (2016) [6], cite. The deterioration rate of a perishable product increases over time and reaches 100% at its sell-by date Sarkar (2012) [7]**.** Some other trend of inventory modelling is about out of stock. In the supermarket, it is observed that, due to the eminence of the retailer, some customers are inclined to await the arrival of the latest stocks if the wait may be brief, at the same time as others may go someplace else. A novel approach was used to model the backlog problem, where consumers are viewed as anxious. Considering different demand patterns, deterioration, shortages, and other parameters; the research work carried out three different scenarios as below the state of art key literature review of research topic mentioned in table 1, table 2 and table 3.

Table 1 Relevant literatures for scenario1 models

Author(s)	**New and buyback used product concept**	**Demand type**	**Deterioration Depends on**	**Shortages Type**	**Model type**
Abad(2001) [8]	No	Linear selling price	Time	Partially Backlog	EOQ
Koh et.al. (2002)	Yes	Constant	-	-	EOQ and

Author(s)	New and buyback used product concept	Demand type	Deterioration Depends on	Shortages Type	Model type
[9]					EPQ
Richter and Dobos (2004) [10]	Yes	Constant	-	-	EOQ
Denial et al.(2003)[11]	Yes	Constant	-	-	Reverse Supply chain
Heese et al.(2005) [12]	Yes	Constant	-	-	EOQ
Kannan et al. (2010) [13]	Yes	Constant	-	-	Supply chain model for battery recycling
Saadany et al.(2010) [14]	Yes	Constant	-	-	EPQ
Sarkar and Sarkar (2013) [15]	No	Stock	Time	Partially and complete backlog cases	EOQ
Soni (2013) [16]	No	Price and inventory level dependent linear demand	Non instantaneous constant	-	EOQ Permissible delay in payment
Chen et.al(2016) [17]	Yes	Constant	-	-	EOQ
Shah and Vaghela (2018) [18]	Yes	Quadratic type demand of time and price for new product, linear demand of time and price for used buyback product.	-	-	EOQ
Sundararajan et al. (2021)[19]	No	Linearly Selling price and exponential time dependent	Non instantaneous constant	Partially Backlog	EOQ
Proposed model-1	Yes	Nonlinear function of price and exponential function of time for New product, linear function of price and time for used buyback product.	-	-	EOQ
Proposed model-2	Yes		Constant	-	EOQ
Proposed model-3	Yes		-	Partially Backlog	EOQ
Proposed model-4	Yes		Constant	Partially Backlog	EOQ

Table 2 Relevant literatures for scenario 2 models

Author(s)	Demand pattern	Deterioration	Carbon Policy	Green Invest-ment	Trade credit financing	Shortages	Model Type
Abad(2001) [8]	Price dependent	Constant	-	-	-	Partially backlog	EOQ
Setak and Daneshfar (2010) [20]	Stock	Constant	-	-	-	Partially backlog	EOQ VMI
Yu et al.	Constant	Constant	-	-	-	-	EOQ

Author(s)	Demand pattern	Deterioration	Carbon Policy	Green Invest-ment	Trade credit financing	Shortages	Model Type
(2012) [21]							VMI
Toptal et al. (2014) [22]	Constant	_	Carbon tax, cap, carbon limit	Yes	-	-	EOQ
Tat et al.(2014) [23]	Constant	Constant	-	-	-	-	EOQ VMI
Taleizadeh et al. (2015) [24]	Price dependent	Constant	-	-	-	-	EOQ VMI
Shah and Jani (2016)[25]	Time dependent quadratic	Time varying	_	_	Yes	-	
Datta(2017) [26]	Selling Price	_	Carbon tax	Yes	_	-	SEPQ
Soni et al. (2018)[27]	Time dependent	Non instantaneous Constant	-	-	-	Partially backlog	EOQ VMI
Bhattacharya and sana (2019) [28]	Random variable and service level	-	-	-	-	-	SEPQ
Zand et al. (2019) [29]	Selling price and greening level	-	-	Yes	-	-	Supply chain
Huang et al.(2020) [30]	Constant	_	Carbon tax	Yes	_	-	EOQ
Sarkar et al.(2020) [31]	Constant	Time varying	_	_	Yes	-	EOQ
Yu et al.(2020) [32]	Selling price and stock	Constant	Carbon tax	-	_	-	EOQ
Mishra et al.(2020) [33]	Constant	-	Carbon tax and cap	-	-	-	SEPQ
Mashud et al.(2020) [34]	Selling price	Constant	Carbon tax	Yes	_	-	EOQ
Shi et al (2020) [35]	Constant	Time Varying (Expiration Date)	Carbon tax, Carbon cap-trade	_	Yes	-	EOQ
Yadav and Khanna(2021) [36]	Selling Price	Time varying (Expiration Date)	Carbon tax	_	-	-	EOQ
Shah et al.(2021) [37]	Stock and selling price	Time varying	_	_	Yes	-	EOQ
Hasan et al.(2021) [38]	green investment and its promotion	_	Carbon tax, Carbon cap-trade, Carbon limit	Yes	_	-	EOQ
Sundararajan	Time	Non	-	-	-	Partially	EOQ

Author(s)	Demand pattern	Deterioration	Carbon Policy	Green Invest-ment	Trade credit financing	Shortages	Model Type
and palanivel (2021) [19]	dependent	instantaneous				backlog	
Paul et al.(2022) [39]	Greening degree and selling price	Constant	Carbon tax	Yes	_	-	SEPQ
Shah et al. (2022) [40]	Stock and selling Price	Constant	Tax and cap	Yes	-	-	SEPQ
Proposed Model-1	Green investment and selling price	Time varying (Expiration Date)	Carbon tax, Carbon cap	Yes	Yes	-	EOQ (Three cases discussed)
Proposed Model-2	Green investment and selling price	Time varying (Expiration Date)	Carbon tax and Cap	Yes	-	-	SEPQ
Proposed Model-3	Green investment and promotion level	Non instantaneous	-	Yes	-	Partially backlog	EOQ (Traditional and VMI model)

Table 3 Relevant literatures for scenario 3 models

Author(s)	Demand Modelling	Deterioration	Greening Investment	Price discount	Markdown strategy	Model Type
Urban and Baker (1997) [41]	Selling price,stock and time	-	-	-	Yes	EOQ
Widyadana and Wee (2007) [42]	Selling price	Constant	-	-	Yes	EOQ
Srivastava and Gupta (2013) [43]	Selling price and time	Delay Constant	-	-	Yes	EPQ
Chen et al.(2016) [44]	Freshness and stock	-	-	-	-	EOQ
Rabbani et al. (2016) [45]	Selling Price	Non instantaneous	-	Yes	-	EOQ
Dobson et al.(2017) [46]	Freshness	-	-	-	-	EOQ
Pal et al.(2018) [47]	Constant	Non instantaneous	-	-	-	EOQ
Raza and Faisal (2018) [48]	Selling price and greening efforts	-	Yes	-	-	EOQ (All units discount)
Mashud et al. (2018)	Stock and Selling price	Non instantaneous	-	-	-	EOQ
Bhaula et al. (2019) [49]	Selling price	Non instantaneous		Yes	-	EOQ
Kamaruzaman and Omar (2020) [50]	Selling price, and stock	Delay Constant	-	-	Yes	EPQ
Mashud et al. (2021) [51]	Selling price	Constant	-	Yes (with advanced payment)	-	EOQ

Author(s)	Demand Modelling	Deterioration	Greening Investment	Price discount	Markdown strategy	Model Type
Agi and Soni (2020) [52]	Selling price, Freshness and stock	Constant	-	-	-	EOQ
Kamaruzaman and Omar (2020) [50]	Selling price, and stock	-	-	-	Yes	
Soni and Shah (2021) [53]	Selling price and freshness	Constant	-	Yes	-	EOQ
Shah et al.(2022a) [54]	Selling price and greening efforts	Constant	Yes	-	-	EOQ (All units discount)
Shah et al.(2022b) [40]	Selling price and greening efforts	Constant	Yes	-	-	EOQ
Shee and Chakrabarti (2022) [55]	Selling price	Time dependent	-	-	Yes	EPQ
Proposed Model-1	**Selling price, freshness and greening efforts**	**Non instantaneous Constant**	**Yes**	**Yes**	-	**EOQ**
Proposed Model-2	**Selling price, freshness and greening efforts**	**Delay constant**	**Yes**	-	**Yes**	**EPQ**

3. Definition of problem and motivation

In the current business trend, consumers are not only interested in purchasing new products from the market, but many are also interested in buying used refurnished, recycled, or repaired products with a price discount because used products are cheaper compared to new products and are sustainable for the environment. According to this hypothesis, the decision-maker is the retailer who not only sells the new products to the consumers but also collects those used products from the consumers and resells them with a price discount. The result is that the retailer generates revenue from the sales of new products as well as used-buyback products. The proposed problem in the EOQ model is defined as the retailer's profit maximization considering various factors.

Carbon emissions from various business activities such as ordering, purchasing, storing, transporting, deterioration of products, environmental impact of the products, etc. are the main factors affecting the sustainability of the environment. Many governments or industries adopted a carbon tax, carbon cap, carbon cap-trade; carbon limits, etc. carbon policies for reducing the effect of carbon emissions from business activities. Investment in green technology is helpful to sustainability. Green technology is an outcome of technological and biological development; it produces less emission and extends the usable life of a product through recycling and reuse. Keeping all these in mind, inventory models for deteriorating items are designed under different carbon policies for controllable carbon emissions by investing in green technology. The proposed problems are (i) an EOQ model for retailers while taking different carbon policies and trade credit payment systems such that total profit is maximum; (ii) a SEPQ model for manufacturers under carbon cap-and-trade policies such that total profit of manufacturers is maximum; (iii) a VMI model and a traditional model in which green investment and their promotion are taken and compared the results of the VMI and traditional models such that total cost of supply chain is minimum.

The freshness and deterioration of the product are major factors that influence market demand. Consumers are moving towards green and sustainable goods. Product freshness is an important component of its quality, and as a result, consumers' purchasing decisions are influenced by the freshness of green products. Greening efforts are the actions taken to minimize the impact trade have on the ecosystem and ensure sustainable products. Taking into account all of these factors, demand for perishable products is a function of the selling price, age of the product (freshness), and greening efforts. Our proposed problems are, to develop the inventory model for non-instantaneous deteriorating perishable products including price discounts such that the total profit of the retailer is maximum and to design an EPQ model for delay deteriorating perishable products with markdown strategy such that the producer's total profit is maximized.

4. Objective and Scope of work

We outline the following as the main objectives of the study and the scope of our work:

4.1 Objectives

- To derive optimal order quantities of new products and used buyback quantity, i.e. to obtain refill policy of new product and take back policy of used product.
- To identify the effect of price discount facility on buyback used product on retailer's profit.

- To identify the effect of the deterioration of new products and used buyback products on total profit.
- To investigate the positive cycle time and shortages period and to derive the impact of a backlogging rate on the total profit of the retailer.
- To optimize the selling price and replenishment cycle time such that the retailer’s total profit is maximized.
- To develop the sustainable EOQ model with green investment and price-dependent demand with carbon tax, cap and trade policy included trade credit financing. To investigate which carbon policy is better for retailers. To identify the role of green technology investment that is helpful to minimize carbon emissions. Investigate the optimum value of selling price, replenishment cycle time and green investment cost such that the retailer’s total profit is maximized.
- To create a SEPQ model for products that incorporates several practical features such as green investment and price-sensitive demand, a time-varying deterioration rate, an expiration date, as well as a carbon cap and tax policy. To derive the optimal value of selling price and green investment cost such that the manufacturer's total profit is maximized.
- To developed the traditional inventory model and VMI model with green investment and its promotion level dependent demand with non instantaneous deterioration and partially backlogged shortages. Obtain results of the traditional inventory model and VMI model, to identify VMI model is better than traditional model. To optimize value of cycle time and green investment cost such that the supply chain total cost is minimized.
- To design and analyze the inventory model considering the selling price, freshness (age of product), and greening efforts related to demand with physical and quality base deterioration, including a selling price discount.
- To optimize the producer’s total profit maximize the optimum value of selling price, cycle time, and markdown percentage considering demand dependent on selling price, freshness (age of product), and greening efforts. To obtain optimum markdown offering time, the quantity of non-deteriorating products and markdown offering quantity.

4.2 Scope of the research work

As we discussed the different literatures related to our proposed work, none has considered the following points: the scope of our proposed research work is to consider the following new ideas, which has not been taken up by any researcher in the study of inventory management.

- The **inventory models of the "new and used buyback products'' concept**
  - a) The demand of new products is a nonlinear form of selling price and exponential form of time. The demand of used buyback product is a linear form of selling price and time.
  - b) The inventory model of new and used buyback products with demand as per a) with deterioration; and rate of deterioration of used buyback product is more than new products.
  - c) The inventory model of new and used buyback products with partially backlogging shortages. The demand during positive cycle time is same as a) and demand during shortages period for new products is non linear function of selling price and for used buyback products is linear function of selling price.
  - d) The inventory model of new and used buyback products with deterioration and partially backlogging shortages.
- **The inventory models with carbon emissions and green investments**
  - e) An EOQ model with price and green investment (as carbon reduction function) dependent under time dependent deterioration rate; Carbon tax, cap-trade carbon policy with trade credit financing.
  - f) A sustainable economic production model with price and green investment (as carbon reduction function) dependent under expiration date; adopted a tax-cap carbon policy.
  - g) The VMI model and traditional model for non-instantaneous deterioration products with green investment (as a carbon reduction function) and promotion level-dependent demand included shortages.
- **The inventory models that incorporated freshness and greening efforts for perishable products**
  - h) The inventory model for non-instantaneous deteriorating perishable products with the concept of greening efforts, freshness and price-related demand, and price discount policy.

i) An EPQ model for delay deteriorating perishable product greening efforts, freshness and price-related demand and markdown strategy.

5. Original contribution by the thesis

The current study highlighted the optimal strategies for deteriorating inventory systems under three different scenarios. Original contribution by thesis can be summarized through the following proposed work.

- Optimal inventory strategies have been proposed for retailer-centric new products and buyback policy for used products, with consideration of deterioration and partially backlogged shortages.
- Retailers' or manufacturers' optimal inventory strategies have been proposed with consideration of carbon emissions and green investments. The work incorporated the different carbon policies, product deterioration, trade credit payment system, shortages, and VMI policy with green investments and price or promotion sensitive demand pattern.
- Optimal inventory strategies for non-instantaneous deteriorating perishable products have been proposed for retailer and producer points, considering product freshness, greening efforts, and selling price-relevant demand patterns. Price discounts and markdown strategies have been adopted for the boost demand.

6. Methodology to derive optimal solution and sensitivity analysis

6.1 Intermediate Value Theorem (Bolzano 1817)

If f a continuous function is on $[\alpha,\beta]$ and $f(\alpha)\neq f(\beta)$, If L is a some number lies between $f(\alpha)$ and $f(\beta)$ then there must be at least $\gamma\in(\alpha,\beta)$ for which $f(\gamma)=L$.

6.2 Optimization problem

Optimization is the technique of achieving the optimal possible result given certain constraints. Several managerial and technological decisions must be made at various phases during the manufacturing, building, structure, and ongoing operations of anything. The final objective of all of these decisions is to maximize (profit) or minimize (cost). The problem of maximization or minimization is called the optimization problem of a mathematical function of one or more variables. The function is known as an objective function. The optimization problem is solved with some limitations or constraints. Single objective non-linear programming problem and Multi-objective non-linear programming problem are main two types of optimization problem.

6.2.1 Single objective non-linear programming problem

The optimization problem have a single objective function is called single objective mathematical programming (SOMP) problem. The minimizing of this type of problem can be expressed as:

$$\begin{aligned}&\text{Determine } z=(z_1,z_2,.....,z_n)^T \qquad (1.1)\\&\text{which minimize } f(z)\\&\text{subject to } z\in X\\&\text{where } X=\{z:g_j(z)\le 0,\ j=1,2,..,m;\ z_j\ge 0,\ i=1,2,..,n\}\end{aligned}$$

Where, $f(z)$ and $g_j(z)$, $j=1,2,..,m$ are defined on n-dimensional set. The objective functions and the constraints are linear in single objective mathematical programming problem,, it become single objective linear programming problem (LPP).

A feasible solution to the problem is a decision variable z which satisfies all the constraint. The problem defined in Eq. (1.1) is to determine a feasible solution z^* such that for each feasible point z, $f(z)\le f(z^*)$ for maximization problem and $f(z)\ge f(z^*)$ for minimization problem and z^* is optimal solution.

6.2.2 Multi-objective non-linear programming problem

Multiple variables cause the problem to become more complicated. Decision-makers find it essential to evaluate the best possible solutions in cases when there are many objectives, taking into account a variety of criteria. The form of multi-objective non linear programming problem is, Determine $z = (z_1, z_2, ..., z_n)^T$ (1.2)

which minimize $F(z) = (f_1(z), f_2(z), ..., f_k(z))^T$

subject to $z \in X$

where $X = \{z : g_j(z) \le 0, j = 1, 2, .., m; z_j \ge 0, i = 1, 2, .., n\}$

where, $f_1(z), f_2(z), ..., f_k(z)$ $(k \ge 2)$ are objectives. It is noted that, if the objectives of the original problem are minimize $f_l(z)$, for $l = 1, 2, .., k_0$ and maximize $f_{k_0+l}(z)$, for $l = k_0 + 1, k_0 + 2, .., k$, then the mathematical formulation of objective is:

Minimize $F(z) = (f_1(z), f_2(z), ..., f_{k_0}(z), -f_{k_0+1}(z), -f_{k_0+2}(z), .., f_k(z))^T$

This is subject to same constraints provided to compute Eq. (1.2).

If $f_l(z), (l = 1, 2, 3.., k)$, and $g_j(z), (j = 1, 2, .., m)$ are linear, the corresponding problem is called Multi Objective Linear Programming Problem. When all or any one of the above functions is non-linear, it is refereed as Multi-Objective Non-Linear Programming Problem.

6.3 Local Minimum

If $z^* \in X$ is said to be a local minima of Eq.(1.1) if there exists $\varepsilon > 0$ such that $f(z) \ge f(z^*), \forall z \in X : \|z - z^*\| < \varepsilon$.

6.4 Convex function

If the Hessian matrix $H(z_1, z_2, ..z_n) = \left[\frac{\partial^2 f}{\partial z_i \partial z_j} \right]_{n \times n}$ is semi-definite/positive definite then a function $f(z_1, z_2, ..z_n)$ is said to be convex function.

6.5 Global minimum

If $f(z) \ge f(z^*)$, $\forall z \in X$ then $z^* \in X$ is said to be a global minimum of Eq.(1.1). Otherwise if the function $f(z)$ is convex then the local minimum solution becomes global minimum.

6.6 Convex programming problem

If the objective function $f(z_1, z_2, ..z_n)$ and the constraints $g_j(z_1, z_2, ..z_n), j = 1, 2, .., m$ are convex then the problem defined in Eq. (1.1) is said to be convex programming problem.

6.7 Solution technique for single-objective problem

6.7.1 Analytical methods

6.7.1.1 Necessary Condition for Optimality

If a function $f(z)$ is defined for all $z \in X$ and has a relative minimum at $z = z^*$, where $z^* \in X$ and all the partial derivatives $\frac{\partial f(z)}{\partial z_p}$ for $p = 1, 2, .., n$ are exists at $z = z^*$ then $\frac{\partial f(z)}{\partial z_p} = 0$.

6.7.1.2 Sufficient Condition for Optimality

The sufficient condition for a stationary point z^* to be an extreme point is that the matrix of second partial derivatives (Hessian Matrix) of $f(z)$ evaluated at $z = z^*$ is

I. Positive definite when z^* a relative minimum point, and
II. Negative is definite when z^* is a relative maximum point.

6.8 Sensitivity analysis of parameters

In this research work, all models are validated by numerical examples and sensitivity analysis of parameters. In sensitivity analysis, the liability of the model is checked, and the sensitivity of parameters is found. Sensitivity analysis is carried out to investigate how different parameters impact the optimal solution of the suggested inventory model by altering each parameter -20% to 20% or -40% to 40% individually while leaving the others unchanged using mathematical software like Maple 18 or Matlab or Mathematica, etc.

7. Achievements with respect to objectives

- This research work given the retailer's ordering and pricing policy for new product and buyback policy of used product such that the total profit of retailer is maximized and importance of price discount on used buyback product in total profit is determined. The deterioration effect on total profit is obtained.
- The design of proposed models for new product and buyback used product with shortages gives the idea to the retailer that decides of proper positive cycle time, shortages period and ordering quantity such that the total profit is maximized.
- The development of the sustainable EOQ model given the optimum value of decision variables such that the retailer's total profit is maximized. The importance of carbon pricing policies and usefulness of trade credit policy is obtained. Investigated that carbon cap-trade policy better than carbon tax policy. The benefits of green investment strategy for environment and profit points of view obtained. The SEPQ model given the optimal value of selling price and green investment cost such that the manufacturer's total profit is maximized. Obtained results of the traditional green inventory model and green VMI model, to identified green VMI model is better than green traditional model.
- The design and analysis of the inventory models considering the selling price, freshness (age of product), and greening efforts related to demand with physical and quality base deterioration helpful to decision maker to find optimum value of selling price, ordering or production quantity, cycle time such that the profit is maximize. Price discount on selling quantity and profit is determined.
- The study has proved helpful for the optimize the producer's total profit maximize at the optimum value of decision variables considering demand dependent on selling price, freshness (age of product), and greening efforts. The research work helpful to obtained optimum markdown offering time, the quantity of non-deteriorating products and markdown offering quantity.

8. Organization of thesis

In this proposed thesis, some real-life inventory problems are formulated and solutions are received. The proposed thesis is divided into **five parts** and **ten chapters**:

Part I: Introduction and literatures review.

This part I contains two chapters: chapter 1 and chapter 2.

Chapter 1: Introduction

The fundamental ideas, basic terminologies, and theories covered in this chapter are those that are utilized most frequently in this area of research.

Chapter 2: Literature Review

Chapter 2 includes the related literatures of proposed research work

Part II: The inventory models of the "new and buyback used products'' concept. (Scenario-1 models)

This part II contains two chapters: chapter 3 and chapter 4.

Chapter 3: Retailer's optimal inventory decisions for new products and a buyback decision for used products

Chapter 3 contained two models:

Model-3.1 O**ptimal inventory decision for non-deteriorating products**

Model-3.2 **Optimal inventory decision for deteriorating products**

Chapter 4: Optimal pricing and replenishment strategies for new products and a buyback strategy of used products from the retailer's points under partial backlog shortages

Chapter 4 contained two models:

Model-4.1 **Optimal inventory strategy for non-deteriorating products for which shortages are partially backlogged**

Model-4.2 **Optimal inventory strategy for deteriorating products for which shortages are partially backlogged**

Part III: The inventory models with carbon emissions and green investments. (Scenario-2 models)

This part III contains three chapters: chapter 5, chapter 6 and chapter 7.

Chapter 5: An EOQ model for deteriorating products with green technology investments and a trade credit payment system

Chapter 6: Sustainable economic production quantity (SEPQ) model for inventory having green technology investments-price sensitive demand with expiration dates

Chapter 7: Optimal green investments and replenishment decisions in vendor-managed inventory systems for non-instantaneous deteriorating products with partial backordering

Part IV: The inventory models that incorporated freshness and greening efforts for perishable products. (Scenario-3 models)

This part IV contains two chapters: chapter 8, chapter 9.

Chapter 8: Optimal greening efforts, pricing and inventory strategies for non instantaneous deteriorating perishable products under price, freshness and green efforts dependent demand with price discount

Chapter 9: An EPQ model for delay deteriorating perishable products with price, freshness and greening efforts dependent demand under markdown strategy

Part V: Summary

Chapter 10: Conclusion and future research scope of the study

9. Conclusion

In our study, the research work carried out three different inventory modelling scenarios: an inventory models with a new and used buyback concept; an inventory models with carbon emissions and green investments; and an inventory models that considered product freshness, greening efforts, a price discount, and a markdown strategy. Inventory modelling of new products have a demand pattern is non linear function of selling price and exponential function of time, linear function of price and time dependent buyback rate and demand rate taken for used product. The demand depended on green investments (as carbon reduction function) and selling price, promotional level of green investment taken in second scenario models, In third scenario models, the demand which depends on products freshness, greening efforts and selling price dependent. Trade credit financing policy, selling price discount policy and markdown policy etc. payment policies adopted for boost the market demand.

These models distinguished under various cases like shortages not allowed or allowed partial backlogging, different type of deterioration rate. Retailer earns revenue of selling new products and buyback used product with considering deterioration and shortages.

Obtained the optimal value of selling price, cycle time such that the retailer's total profit is maximized. The different sources of carbon emissions to be considered, green technology investment and their effects on reducing carbon emissions are discussed. Optimize the selling price and green investments subject to maximize the profit of retailer or manufactures. The VMI and tradition inventory models are discussed with green investments strategy. Results shows VMI policy is beneficial in comparison of individual supply chain policy. Perishable product freshness, greening efforts, and selling price require more attention in inventory modelling. Price discount or markdown strategy to be applied during deteriorating period for increases the selling of perishable products at end of inventory period in third scenario models. Optimal value of selling price, greening efforts, markdown percentages, markdown offering period, and markdown ordered quantity helps to retailer or producer to maximize their profit.

All model findings are shown analytically and graphically. The models can be used by operation research and inventory professionals to solve their present manufacturing and stock management challenges.

10. Copies of papers published and a list of all publications arising from the thesis

- **Published research papers**

1. Katariya DK, Shukla KT. "Retailer's Ordering and Pricing Strategy for New Product and Buyback Strategy for Used Product with Deterioration". *International Journal of Operations Research*. 2021, Jun 1; 18(2):17-28. **(UGC care listed Journal)**
2. Katariya DK, Shukla KT. '' Retailer's optimal inventory strategy for new product and buy back strategy for used product''. *Advances and Applications in Mathematical Sciences.* 2021, Sept 1; 20(11): pp.2917-2936. **(UGC care listed Journal)**
3. Katariya D, Shukla K. ''Sustainable economic production quantity (SEPQ) model for inventory having green technology investments-price sensitive demand with expiration dates''. *Economic Computation & Economic Cybernetics Studies & Research.* 2022 Jul 1; 56(3).pp. 135-152. **(SCI Journal)**

- **Accepted research papers in different journals**

1. Katariya D, Shukla K. ‘’Pricing and Ordering Strategy for New Product and Buyback Strategy for Used Product from Retailer’s Point’’ *Int. J. of Operational Research,* Accepted in August 2021. **(Scopus index Journal).**
2. Katariya D, Shukla K. ‘’An EOQ model for Deteriorating Products with Green Technology Investment and Trade Credit Financing’’ *Int. J. of Procurement Management,* Accepted in June 2022. **(Scopus index Journal).**
3. Katariya D, Shukla K. ‘’Retailer’s Optimal Pricing and Replenishment Policy for New Product and Buy Back Policy for Used Product with Price and Time Dependent Demand and Shortages’’. *TARU Journal of Organizational Behaviour & Analytics,* Accepted in Feb 2023. **(Peer Review Journal).**
4. Katariya D, Shukla K. ‘’Optimal Greening efforts, Pricing and Inventory Strategies for Non Instantaneous Deteriorating Perishable Products under Price, Freshness and Green efforts Dependent demand with Price Discount’’. *Int. J. Of Mathematics in Operational Research,* Accepted in April 2023. **(Scopus index Journal).**

- **Research papers under communication:**

1. Katariya D, Shukla K.’’ Optimal Green Investments and Replenishment Decisions in Vendor Managed Inventory System for Non Instantaneous Deteriorating Products with Partially Backordering’’. *Int. J. of Business Performance and Supply Chain Modelling,* **(Scopus index Journal).**
2. Katariya D, Shukla K. ‘’An EPQ model for delay deteriorating products with price, freshness and greening efforts dependent demand under Markdown Strategy’’. *Yugoslav Journal of Operations Research,* **(Scopus index Journal)**

- **Accepted book chapter:**

1. Katariya D, Shukla K. ‘’A sustainable inventory models under expiration dates, green technology investments and trade credit policy’’. *Supply Chain Finance Modelling and Optimization,* Accepted in May 2023. **(Springer Nature Pvt. Ltd.)**

- **Research papers presented in conferences:**

1. A paper titled, ''A Sustainable inventory model of perishable products under expiration dates and green technology investments'', presented in the 4th *International conference on Frontiers Industrial and Applied Mathematics*, organized by SLIET Longowal, Punjab, India, 21-22 December 2021**.**
2. A paper titled,'' Optimal Strategies for Vendor managed Inventory Model with Green investment and it's promotion level dependent demand and shortages '', presented in *the International Conference on Applied Mathematical Sciences-2022,* jointly organized by Department of Mathematics, Gujarat University and Department of Applied Science and Humanities, Parul University, Vadodara, Gujarat, India, 12-13 November 2022.

11. References

[1] J. M. Opher Baronr and Iman Hajizadeh, "Now playing: DVD purchasing for a multilocation rental firm," *Manuf. Serv. Oper.*, vol. 13, no. 2, pp. 209–226, doi: https://doi.org/10.1287/msom.1100.0314.

[2] P. Ghare and G. Schrader, "An inventory model for exponentially deteriorating items," *Journal of Industrial Engineering*, vol. 14, no. 2, pp. 238–243, 1963.

[3] Y. S. NH Shah, "Literature survey on inventory models for deteriorating items," *EKONOMSKI ANALI*, vol. 44, no. 145, pp. 221–237, 2020.

[4] S. K. Goyal and B. C. Giri, "Recent trends in modeling of deteriorating inventory," *European Journal of operational research*, vol. 134, no. 1, pp. 1–16, 2001.

[5] M. Bakker, J. Riezebos, and R. H. Teunter, "Review of inventory systems with deterioration since 2001," *European journal of operational research*, vol. 221, no. 2, pp. 275–284, 2012.

[6] L. Janssen, T. Claus, and J. Sauer, "Literature review of deteriorating inventory models by key topics from 2012 to 2015," *International Journal of Production Economics*, vol. 182, pp. 86–112, 2016.

[7] B. Sarkar, "An EOQ model with delay in payments and time varying deterioration rate," *Mathematical and Computer Modelling*, vol. 55, no. 3–4, pp. 367–377, 2012.

[8] P. L. Abad, "Optimal price and order size for a reseller under partial backordering," *Computers & Operations Research*, vol. 28, no. 1, pp. 53–65, 2001.

[9] S.-G. Koh, H. Hwang, K.-I. Sohn, and C.-S. Ko, "An optimal ordering and recovery policy for reusable items," *Computers & Industrial Engineering*, vol. 43, no. 1–2, pp. 59–73, 2002.

[10] K. Richter and I. Dobos, "Production-inventory control in an EOQ-type reverse logistics system," *Supply chain management and reverse logistics*, pp. 139–160, 2004.

[11] S. E. Daniel, C. P. Pappis, and T. G. Voutsinas, "Applying life cycle inventory to reverse supply chains: a case study of lead recovery from batteries," *Resources, Conservation and Recycling*, vol. 37, no. 4, pp. 251–281, 2003.

[12] H. S. Heese, K. Cattani, G. Ferrer, W. Gilland, and A. V. Roth, "Competitive advantage through take-back of used products," *European Journal of Operational Research*, vol. 164, no. 1, pp. 143–157, 2005.

[13] G. Kannan, P. Sasikumar, and K. Devika, “A genetic algorithm approach for solving a closed loop supply chain model: A case of battery recycling,” *Applied mathematical modelling*, vol. 34, no. 3, pp. 655–670, 2010.
[14] A. M. El Saadany and M. Y. Jaber, “A production/remanufacturing inventory model with price and quality dependant return rate,” *Computers & Industrial Engineering*, vol. 58, no. 3, pp. 352–362, 2010.
[15] B. Sarkar and S. Sarkar, “An improved inventory model with partial backlogging, time varying deterioration and stock-dependent demand,” *Economic Modelling*, vol. 30, pp. 924–932, 2013.
[16] H. N. Soni, “Optimal replenishment policies for non-instantaneous deteriorating items with price and stock sensitive demand under permissible delay in payment,” *International journal of production Economics*, vol. 146, no. 1, pp. 259–268, 2013.
[17] C.-K. Chen, T.-C. Weng, and C.-C. Lo, “Optimal replenishment quantity for new products and return rate of used products for a retailer,” *Applied Mathematical Modelling*, vol. 40, no. 23–24, pp. 9754–9766, 2016.
[18] N. H. Shah and C. R. Vaghela, “Retailer’s optimal pricing and replenishment policy for new product and optimal take-back quantity of used product,” *Yugoslav Journal of Operations Research*, vol. 28, no. 3, pp. 345–353, 2018.
[19] R. Sundararajan, M. Palanivel, and R. Uthayakumar, “An EOQ model of non-instantaneous deteriorating items with price, time-dependent demand and backlogging,” *Journal of Control and Decision*, vol. 8, no. 2, pp. 135–154, 2021.
[20] L. Daneshfar and M. Setak, “An inventory model for deteriorating items using vendor-managed inventory policy,” *International Journal of Engineering*, vol. 27, no. 7, pp. 1081–1090, 2014.
[21] Y. Yu, Z. Wang, and L. Liang, “A vendor managed inventory supply chain with deteriorating raw materials and products,” *International journal of production economics*, vol. 136, no. 2, pp. 266–274, 2012.
[22] A. Toptal, H. Özlü, and D. Konur, “Joint decisions on inventory replenishment and emission reduction investment under different emission regulations,” *International journal of production research*, vol. 52, no. 1, pp. 243–269, 2014.
[23] R. Tat, A. A. Taleizadeh, and M. Esmaeili, “Developing economic order quantity model for non-instantaneous deteriorating items in vendor-managed inventory (VMI) system,” *International Journal of Systems Science*, vol. 46, no. 7, pp. 1257–1268, 2015.
[24] A. A. Taleizadeh, M. Noori-daryan, and L. E. Cárdenas-Barrón, “Joint optimization of price, replenishment frequency, replenishment cycle and production rate in vendor managed inventory system with deteriorating items,” *International Journal of Production Economics*, vol. 159, pp. 285–295, 2015.
[25] N. H. Shah, U. Chaudhari, and M. Y. Jani, “Optimal policies for time-varying deteriorating item with preservation technology under selling price and trade credit dependent quadratic demand in a supply chain,” *International Journal of Applied and Computational Mathematics*, vol. 3, no. 2, pp. 363–379, 2017.
[26] TK Datta, “Effect of green technology investment on a production-inventory system with carbon tax,” *Advances in Operations Research*, pp. 1–12, 2017, doi: https://doi.org/10.1155/2017/4834839.
[27] A. S. G. Hardik N. Soni and Hetal R. Patel, “Vendor managed inventory model for non-instantaneous deteriorating product with quadratic demand allowing partial backlogging,” *Uncertain Supply Chain Management*, vol. 6, pp. 321–334, 2018.

[28] M. Bhattacharyya and S. S. Sana, "A mathematical model on eco-friendly manufacturing system under probabilistic demand," *RAIRO-operations Research*, vol. 53, no. 5, pp. 1899–1913, 2019.
[29] F. Zand, S. Yaghoubi, and S. J. Sadjadi, "Impacts of government direct limitation on pricing, greening activities and recycling management in an online to offline closed loop supply chain," *Journal of cleaner production*, vol. 215, pp. 1327–1340, 2019.
[30] Y.-S. Huang, C.-C. Fang, and Y.-A. Lin, "Inventory management in supply chains with consideration of Logistics, green investment and different carbon emissions policies," *Computers & Industrial Engineering*, vol. 139, p. 106207, 2020.
[31] B. Sarkar, B. K. Dey, M. Sarkar, S. Hur, B. Mandal, and V. Dhaka, "Optimal replenishment decision for retailers with variable demand for deteriorating products under a trade-credit policy," *RAIRO-Operations Research*, vol. 54, no. 6, pp. 1685–1701, 2020.
[32] C. Yu, Z. Qu, T. W. Archibald, and Z. Luan, "An inventory model of a deteriorating product considering carbon emissions," *Computers & Industrial Engineering*, vol. 148, p. 106694, 2020.
[33] U. Mishra, J.-Z. Wu, and B. Sarkar, "A sustainable production-inventory model for a controllable carbon emissions rate under shortages," *Journal of Cleaner Production*, vol. 256, p. 120268, 2020.
[34] A. H. Md Mashud, M. R. Hasan, H. M. Wee, and Y. Daryanto, "Non-instantaneous deteriorating inventory model under the joined effect of trade-credit, preservation technology and advertisement policy," *Kybernetes*, vol. 49, no. 6, pp. 1645–1674, 2020.
[35] Y. Shi, Z. Zhang, S.-C. Chen, L. E. Cárdenas-Barrón, and K. Skouri, "Optimal replenishment decisions for perishable products under cash, advance, and credit payments considering carbon tax regulations," *International Journal of Production Economics*, vol. 223, p. 107514, 2020.
[36] S. Yadav and A. Khanna, "Sustainable inventory model for perishable products with expiration date and price reliant demand under carbon tax policy," *Process Integration and Optimization for Sustainability*, vol. 5, pp. 475–486, 2021.
[37] N. H. Shah, K. Rabari, and E. Patel, "An inventory model with stock-dependent demand under trade-credit policy and fixed life-time," *International Journal of Procurement Management*, vol. 14, no. 2, pp. 264–275, 2021.
[38] M. R. Hasan, T. C. Roy, Y. Daryanto, and H.-M. Wee, "Optimizing inventory level and technology investment under a carbon tax, cap-and-trade and strict carbon limit regulations," *Sustainable Production and Consumption*, vol. 25, pp. 604–621, 2021.
[39] A. Paul, M. Pervin, S. K. Roy, N. Maculan, and G.-W. Weber, "A green inventory model with the effect of carbon taxation," *Annals of Operations Research*, vol. 309, no. 1, pp. 233–248, 2022.
[40] N. Shah, P. Shah, and M. Patel, "Pricing Decisions with Effect of Advertisement and Greening Efforts for a Greengocer," *Sustainability*, vol. 14, no. 21, p. 13807, 2022.
[41] T. L. Urban and R. Baker, "Optimal ordering and pricing policies in a single-period environment with multivariate demand and markdowns," *European Journal of Operational Research*, vol. 103, no. 3, pp. 573–583, 1997.
[42] H. M. Wee and G. A. Widyadana, "A replenishment policy for item with price dependent demand and deteriorating under markdown policy," *Jurnal Teknik Industri*, vol. 9, no. 2, pp. 75–84, 2007.
[43] M. Srivastava and R. Gupta, "An EPQ model for deteriorating items with time and price dependent demand under markdown policy," *Opsearch*, vol. 51, pp. 148–158, 2014.

[44] S.-C. Chen, J. Min, J.-T. Teng, and F. Li, “Inventory and shelf-space optimization for fresh produce with expiration date under freshness-and-stock-dependent demand rate,” *Journal of the Operational Research Society*, vol. 67, no. 6, pp. 884–896, 2016.
[45] M. Rabbani, N. P. Zia, and H. Rafiei, “Joint optimal dynamic pricing and replenishment policies for items with simultaneous quality and physical quantity deterioration,” *Applied Mathematics and Computation*, vol. 287, pp. 149–160, 2016.
[46] G. Dobson, E. J. Pinker, and O. Yildiz, “An EOQ model for perishable goods with age-dependent demand rate,” *European Journal of Operational Research*, vol. 257, no. 1, pp. 84–88, 2017.
[47] H. Pal, S. Bardhan, and B. Giri, “Optimal replenishment policy for non-instantaneously perishable items with preservation technology and random deterioration start time,” *International Journal of Management Science and Engineering Management*, vol. 13, no. 3, pp. 188–199, 2018.
[48] S. A. Raza and M. N. Faisal, “Inventory models for joint pricing and greening effort decisions with discounts,” *Journal of Modelling in Management*, vol. 13, no. 1, pp. 2–26, 2018.
[49] B. Bhaula, J. K. Dash, and M. Rajendra Kumar, “An optimal inventory model for perishable items under successive price discounts with permissible delay in payments,” *Opsearch*, vol. 56, pp. 261–281, 2019.
[50] N. A. Kamaruzaman and M. Omar, “Inventory model for a fresh product when demand depends on freshness, price, inventory level and expiration date under markdown policy,” *Journal of Intelligent & Fuzzy Systems*, vol. 39, no. 3, pp. 3453–3461, 2020.
[51] A. H. M. Mashud, D. Roy, Y. Daryanto, and H.-M. Wee, “Joint pricing deteriorating inventory model considering product life cycle and advance payment with a discount facility,” *RAIRO-operations Research*, vol. 55, pp. S1069–S1088, 2021.
[52] M. A. Agi and H. N. Soni, “Joint pricing and inventory decisions for perishable products with age-, stock-, and price-dependent demand rate,” *Journal of the Operational Research Society*, vol. 71, no. 1, pp. 85–99, 2020.
[53] H. N. Soni, “Joint Pricing and Inventory Policies for Perishable Items with Price Discount based on Freshness Index,” *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, vol. 12, no. 11, pp. 1954–1963, 2021.
[54] N. H. Shah, K. Rabari, and E. Patel, “Greening efforts and deteriorating inventory policies for price-sensitive stock-dependent demand,” *International Journal of Systems Science: Operations & Logistics*, vol. 10, no. 1, p. 2022808, 2023.
[55] S. Shee and T. Chakrabarti, “An EPQ Model for Imperfect Production System with Deteriorating Items, Price-Dependent Demand, Rework and Lead Time Under Markdown Policy,” in *Data Analytics and Artificial Intelligence for Inventory and Supply Chain Management*, Springer, 2022, pp. 81–96.